Thresholding Using Two-Dimensional Histogram and Fuzzy Entropy Principle


Abstract—This paper presents a thresholding approach by performing fuzzy partition on a two-dimensional (2-D) histogram based on fuzzy relation and maximum fuzzy entropy principle. The experiments with various gray level and color images have demonstrated that the proposed approach outperforms the 2-D nonfuzzy approach and the one-dimensional (1-D) fuzzy partition approach.

Index Terms—Fuzzy region, fuzzy relations, maximum fuzzy entropy principle, threshold, 2-D histogram.

I. INTRODUCTION

Image thresholding is an important technique for image processing and pattern recognition, which is also regarded as the first step for image understanding. Many methods have been proposed to select the thresholds automatically [1]. Most bilevel thresholding techniques can be extended to the case of multithresholding, therefore, we focus on the bilevel thresholding techniques in this paper. The proposed approach will automatically determine the fuzzy region and find the thresholds based on the maximum fuzzy entropy principle. It involves a novel fuzzy partition on a two-dimensional (2-D) histogram where a 2-D fuzzy entropy is defined, and a genetic algorithm is employed to find the optimal result.

II. PROPOSED APPROACH

In order to obtain a 2-D histogram of an image, we define the local average of a pixel, \( f(x, y) \), as the average intensity of its four neighbors denoted by \( g(x, y) \).

\[
g(x, y) = \left[ \frac{1}{4} f(x, y + 1) + f(x, y - 1) + f(x + 1, y) + f(x - 1, y) \right] + 0.5. \tag{1}\]

A 2-D histogram is an array \((L \times L)\) with the entries representing the number of occurrences of the pair \((f(x, y), g(x, y))\). A 2-D histogram can be viewed as a full Cartesian product of two sets \(X\) and \(Y\), where \(X\) represents the gray levels and \(Y\) represents the local average gray levels: \(X = Y = \{0, 1, 2, \cdots, L-1\}\). The pixels having the same intensity but different spatial features can be distinguished in the second dimension (local average gray level) of a 2-D histogram.

The Block\(_X\) and Block\(_Y\) are defined in Fig. 1(a) and (b), respectively. Four fuzzy sets, Bright\(_X\), Dark\(_X\), Bright\(_Y\), and Dark\(_Y\), are defined based on the \(S\)-function and the corresponding \(Z\)-function [2], [3] as follows:

\[
\text{Bright}_X = \sum_{x \in X} \frac{\mu_{\text{Bright}_X}(x)}{x} = \sum_{x \in X} \frac{S(x, a, b, c)}{x} \\
\text{Dark}_X = \sum_{x \in X} \frac{\mu_{\text{Dark}_X}(x)}{x} = \sum_{x \in X} \frac{Z(x, a, b, c)}{x}
\]

REFERENCES

The following four entropies can be computed:

\[ H_{\text{entropy}}(R_B) = - \sum_{(x, y) \in R_B} \frac{n_{xy}}{\sum_{(x, y) \in R_B} n_{xy}} \log \frac{n_{xy}}{\sum_{(x, y) \in R_B} n_{xy}} \]  

\[ H_{\text{fuzzy}}(R_B) = - \sum_{(x, y) \in R_B} \frac{n_{xy}}{\sum_{(x, y) \in R_B} n_{xy}} \log \frac{n_{xy}}{\sum_{(x, y) \in R_B} n_{xy}} \]  

\[ H_{\text{entropy}}(R_D) = - \sum_{(x, y) \in R_D} \frac{n_{xy}}{\sum_{(x, y) \in R_D} n_{xy}} \log \frac{n_{xy}}{\sum_{(x, y) \in R_D} n_{xy}} \]  

\[ H_{\text{fuzzy}}(R_D) = - \sum_{(x, y) \in R_D} \frac{n_{xy}}{\sum_{(x, y) \in R_D} n_{xy}} \log \frac{n_{xy}}{\sum_{(x, y) \in R_D} n_{xy}} \]  

where \( n_{xy} \) is the element in the 2-D histogram which represents the number of occurrences of the pair \((x, y)\). The membership functions \( \mu_{\text{Height}}(x, y) \) and \( \mu_{\text{Dark}}(x, y) \) are defined in (2) and (3), respectively. It should be noticed that the probability computations \( n_{xy}/\sum n_{xy} \) in the four regions are independent of each other.

To find the best set of \( a, b, \) and \( c \) is an optimization problem which can be solved by: heuristic searching, simulated annealing, genetic algorithm, etc. In this paper, we use genetic algorithm [5] to search for the optimal solution. The proposed method consists of the following three major steps:

1) find the 2-D histogram of the image;
2) perform fuzzy partition on the 2-D histogram;
3) compute the fuzzy entropy.

Step 1 needs to be executed only once while steps 2) and 3) are performed iteratively for each set of \((a, b, c)\). The optimum \((a, b, c)\) determines the fuzzy region (i.e., interval \([a, c]\)). The threshold is selected as the crossover point of the membership function which has membership 0.5 implying the largest fuzziness.

Once the threshold vector \((s, t)\) is obtained, it divides the 2-D histogram into four blocks, i.e., a dark block \(B_1\), a bright block \(B_2\), and two noise (edge) blocks, \(B_3\) and \(B_4\), as shown in Fig. 1(c). The bright extraction method is expressed as [6]

\[ f_{s, t}(x, y, \text{bright}) = \begin{cases} g_1, & f(x, y) \geq t \land g(x, y) \geq s \\ g_0, & \text{otherwise} \end{cases} \]  

Conversely, the dark portion extraction is

\[ f_{s, t}(x, y, \text{dark}) = \begin{cases} g_0, & f(x, y) < t \land g(x, y) < s \\ g_1, & \text{otherwise} \end{cases} \]  

### III. EXPERIMENTAL RESULTS AND DISCUSSIONS

Most gray level image thresholding techniques can be extended to color images by directly processing each component of a color space, then combining the results in some way to obtain the final image. For a color image, we apply the proposed approach to RGB components of the color space, respectively, and then combine the three results into a new RGB color image.

We test the proposed approach on many monochrome and color images. We only show three figures here. The gray level of monochrome images and RGB components of color images ranges from 0 to 255. For a monochrome image, the corresponding bilevel threshold image is expressed in two intensities 0 and 255. For a color image, the bilevel threshold image of each component (R, G, and B) is expressed in two
Fig. 2. (a) Original image, (b) result using 1-D fuzzy approach, (c) result using 2-D nonfuzzy approach, and (d) result using proposed 2-D fuzzy approach.

Fig. 3. (a) The original image, (b) result using 1-D fuzzy approach, (c) result using 2-D nonfuzzy approach, and (d) result using proposed 2-D fuzzy approach.

Intensities: $g_0$ and $g_1$, where $g_0$ represents the gray level with the maximal number of pixels which are less than the threshold, and $g_1$, the gray level with the maximal number of pixels which are greater than the threshold. Then these bilevel images are combined to a RGB color image.

In order to show the importance of spatial information in threshold selection, we compare the results of the proposed approach with the ones of the 1-D fuzzy approach which uses fuzzy $c$-partition and maximum fuzzy entropy principle to select thresholds [6]. The results of the proposed approach are also compared with those of the 2-D nonfuzzy (crisp) approach to demonstrate the power of fuzzy set theory. The 2-D nonfuzzy approach uses 2-D histogram and maximum entropy principle to select thresholds [7]. For Fig. 2, the threshold vectors are (119, 159) and (112, 112) by using 2-D nonfuzzy approach and 2-D fuzzy approach, respectively. The threshold value is 127 by using 1-D fuzzy approach. Fig. 2(d) has much clearer features of image than those in Fig. 2(b) and (c). The sky and the tower are better segmented in Fig. 2(d) than that in Fig. 2(b) and (c). For Fig. 3(b), the threshold values are 102, 113, and 112 for R, G, and B components, respectively. For Fig. 3(c), the threshold vectors are (83, 82), (81, 75), and (66, 69) for R, G, and B components, respectively. For Fig. 3(d), the threshold vectors are (81, 81), (100, 100), and (154, 154) for R, G, and B components respectively. Fig. 3(d) is the only one discriminating the blue sky from the cornfield, and the tractor in Fig. 3(d) is much better extracted than that in Fig. 3(b) and (c). The upper-right corner of Fig. 3(d) was misclassified which was caused by bilevel thresholding. This also happened in Fig. 3(c). However, Fig. 3(d) has the best result. For Fig. 4(b), the threshold values are 252, 211, and 164 for R, G, and B components, respectively. For Fig. 4(c), the threshold vectors are (138, 128), (164, 196), and (180, 182) for R, G, and B components, respectively. For Fig. 4(d), the threshold vectors are (215, 215), (196, 196), and (171, 171) for R, G, and B components, respectively. In Fig. 4(d), the eyes, nose, and mouth, are well extracted, and the colors of the dress and hair are different. In Fig. 4(b), the colors of the dress,
A novel 2-D fuzzy partition characterized by parameters $a$, $b$, and $c$ is proposed which divides a 2-D histogram into two fuzzy subsets “dark” and “bright.” For each fuzzy subset, one fuzzy entropy and one nonfuzzy entropy are defined based on the fuzziness of the regions. The best fuzzy partition was found based on the maximum fuzzy entropy principle, and the corresponding parameters $a$, $b$, and $c$ determines the fuzzy region $[a, c]$. The threshold is selected as the crossover point of the fuzzy region. The experimental results show that the spatial information of pixels should be considered in the selection of thresholds and the 2-D fuzzy approach outperforms the 2-D crisp approach and the 1-D fuzzy approach.

### IV. Conclusions

A novel 2-D fuzzy partition characterized by parameters $a$, $b$, and $c$ is proposed which divides a 2-D histogram into two fuzzy subsets “dark” and “bright.” For each fuzzy subset, one fuzzy entropy and one nonfuzzy entropy are defined based on the fuzziness of the regions. The best fuzzy partition was found based on the maximum fuzzy entropy principle, and the corresponding parameters $a$, $b$, and $c$ determines the fuzzy region $[a, c]$. The threshold is selected as the crossover point of the fuzzy region. The experimental results show that the spatial information of pixels should be considered in the selection of thresholds and the 2-D fuzzy approach outperforms the 2-D crisp approach and the 1-D fuzzy approach.

### TABLE I

<table>
<thead>
<tr>
<th>Images</th>
<th>Size</th>
<th>1D fuzzy</th>
<th>2D Nonfuzzy</th>
<th>2D fuzzy</th>
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<tbody>
<tr>
<td>Tower</td>
<td>512×768</td>
<td>3 sec.</td>
<td>4 sec.</td>
<td>13 sec.</td>
</tr>
<tr>
<td>Cornfield</td>
<td>256×256</td>
<td>8 sec.</td>
<td>3 sec.</td>
<td>14 sec.</td>
</tr>
<tr>
<td>Girl</td>
<td>256×256</td>
<td>7 sec.</td>
<td>3 sec.</td>
<td>14 sec.</td>
</tr>
</tbody>
</table>

### REFERENCES


### Regions Adjacency Graph Applied to Color Image Segmentation

Alain Trémeau and Philippe Colantoni

**Abstract**—The aim of this paper is to present different algorithms, based on a combination of two structures of graph and of two color image processing methods, in order to segment color images. The structures used in this study are the region adjacency graph and the line graph associated. We will see how these structures can enhance segmentation processes such as region growing or watershed transformation. The principal advantage of these structures is that they give more weight to adjacency relationships between regions than usual methods. Let us note nevertheless that this advantage leads in return to adjust more parameters than other methods to best refine the result of the segmentation. We will show that this adjustment is necessarily image dependent and observer dependent.

**Index Terms**—Color image segmentation, region adjacency graph, region growing process, watershed process.

### I. Introduction

Even if many algorithms are available for color image segmentation [1]–[7], the literature is not as rich as that for grey level images [7], especially when we refer to segmentation algorithms based on region growing processes. Yet, it has been demonstrated that, for several sets of images, region growing processes best perform than clustering or thresholding approaches because they deal with spatial repartition of color information. Region growing algorithms typically start with seed pixels, then iteratively add to regions unassigned neighboring pixels which satisfy one or several homogeneity criteria. Thus, we can define a region as being a set of connected pixels which satisfy some homogeneity criteria. Several criteria linked to color similarity or spatio-color similarity can be used to analyze if a pixel belongs or not to a region [8], [9]. These criteria can be defined according to local, regional and global relationships. In a previous approach, we have shown that three criteria of homogeneity must be used to perform a relevant segmentation [10]. These criteria are as follows:

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