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A model of a trust-based recommendation system on a social network

Frank Edward Walter · Stefano Battiston · Frank Schweitzer

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Abstract In this paper, we present a model of a trust-based recommendation system on a social network. The idea of the model is that agents use their social network to reach information and their trust relationships to filter it. We investigate how the dynamics of trust among agents affect the performance of the system by comparing it to a frequency-based recommendation system. Furthermore, we identify the impact of network density, preference heterogeneity among agents, and knowledge sparseness to be crucial factors for the performance of the system. The system self-organises in a state with performance near to the optimum; the performance on the global level is an emergent property of the system, achieved without explicit coordination from the local interactions of agents.

Keywords Recommender system · Trust · Social network

1 Introduction and motivation

In recent years, the Internet has become of greater and greater importance in everyone's life. People use their computers for communication with others, to buy and sell products on-line, to search for information, and to carry out many more tasks. The Internet has become a social network, "linking people, organisations, and knowledge" [33] and it has taken the role of a platform on which people pursue an increasing amount of activities that they have usually only done in the real-world.

This development confronts people with an *information overload*: they are facing too much data to be able to effectively filter out the pieces of information that are most appropriate

F. E. Walter (✉) · S. Battiston · F. Schweitzer
Chair of Systems Design, ETH Zurich, Kreuzplatz 5, 8032 Zurich, Switzerland
e-mail: fewalter@ethz.ch

S. Battiston
e-mail: sbattiston@ethz.ch

F. Schweitzer
e-mail: fschweitzer@ethz.ch

for them. The exponential growth of the Internet [21] implies that the amount of information accessible to people grows at a tremendous rate. Historically, people have—in various situations—already had to cope with information overload and they have intuitively applied a number of *social mechanisms* that help them deal with such situations. However, many of these, including the notion of trust, do not yet have an appropriate *digital mapping* [23]. Finding suitable representations for such concepts is a topic of on-going research [1,10,17,23,27,29].

The problem of information overload has been in the focus of recent research in computer science and a number of solutions have been suggested. The use of search engines [9] is one approach, but so far, they lack personalisation and usually return the same result for everyone, even though any two people may have vastly different profiles and thus be interested in different aspects of the search results. A different proposed approach are recommendation systems [24–26,30].

In the following, we present a model of a trust-based recommendation system which, in an automated and distributed fashion, filters information for agents based on the agents' social network and trust relationships [6,15,19,25].

Trust is a topic which has recently been attracting research from many fields, including, but not limited to, computer science, cognitive sciences, sociology, economics, and psychology. As a result of this, there exists a plethora of definitions of trust, some similar to each other, some different from each other. In the context of our model, trust can be defined as the *expectancy of an agent to be able to rely on some other agent's recommendations*.

There are many areas of application in which such systems, or similar ones, are applicable: some obvious examples would be the facilities to share opinions and/or ratings that many shopping or auctioning web sites offer, but the same principles of combining social networks and trust relationships can be applied in other domains as well: for example, in the scientific community, in form of a recommendation system for journal, conference, and workshop contributions.

The model that we are going to present enables a quantitative study of the problem and also provides a sketch for a solution in terms of a real Internet application/web service. The idea at the core of the model is that agents

- leverage their social network to *reach information*; and
- make use of trust relationships to *filter information*.

In the following, we describe the model and the results obtained by simulating the model with multi-agent simulations. To some extent, it is also possible to make analytical predictions of the performance of the system as a function of the preferences of the agents and the structure of the social network.

The remainder of the paper is organised as follows: in the following section, we put our work into the context of the related work. Then, we present our model of a trust-based recommendation system on a social network. This is followed by an analysis of the results from computer simulations and analytical approximations of the model. Subsequently, we illustrate a number of possible extensions and conclude with a summary of the work.

2 Related work

Recent research in computer science has dealt with recommendation systems [26,30]. Such systems mostly fall into two classes: content-based methods suggest items by matching agent profiles with characteristics of products and services, while collaborative filtering methods measure the similarity of preferences between agents and recommend what similar agents have already chosen [30]. Often, recommendation systems are centralised and, moreover, they are offered by entities which are not independent of the products or services that they provide recommendations on—often, this constitutes a bias or conflict-of-interest.

Trust is a topic which is of ubiquitous importance to people. This is why it has been studied in many disciplines, among them computer science, cognitive sciences, sociology, economics, and psychology [1,10,17,23,29]. In computer science, trust was initially seen as a method to enhance security systems [17]: cryptography allows to ensure the authenticity, confidentiality, and integrity, of the communication between two parties Alice and Bob, but it does not allow Alice to judge how trustworthy Bob is, and vice versa. In such contexts, trust has often been formalised with logical models [20,22]. For a more detailed overview of trust in the literature, please refer to [29].

Additionally, the diffusion of information technologies in business and social activities results in intricate networks of electronic relationships. In particular, many economic activities via electronic transactions require the presence of or benefit from a system of trust and distrust in order to ensure the fulfilment of contracts [23,29]. However, trust plays a crucial role not only by supporting the security of contracts between agents, but also because agents rely on the expertise of other trusted agents in their decision-making.

Along these lines, some recent works have suggested to combine distributed recommendation systems with trust and reputation mechanisms [14,24,25]. It is because of the fact that both building expertise and testing items available on the market are costly activities, individuals in the real world attempt to reduce such costs through the use of their social/professional networks.

Such complex networks, in particular their structure and function, are the subject of an extensive and growing body of research across disciplines [28]. Social networks have received special attention [5] and it has been shown that their structure plays an important role in decision-making processes [4,7,13].

With respect to existing models of trust-based recommender systems operating on social networks in the literature [14,19,25], the contributions of our work are the following: we provide analytical results for the performance of the system within a range of network density, preference heterogeneity among agents, and knowledge sparseness. We also report on extensive multi-agent simulations supporting our predictions. The notion of trust that we use is quite general because it relies on the utility of an agent from interacting with other agents. Thus, it can be extended to represent more than just the similarity of preferences between two agents [16,34]. With respect to [25], besides the above, our model includes a mechanism for propagation of trust along paths in the social network. Finally, we provide a framework which allows the study of two crucial aspects, evolution and robustness, both from an analytical point-of-view, but also by multi-agent simulations; in this respect, the framework could be validated against empirical data along the lines of [24].

3 Model description

The model deals with agents which have to decide for a particular item that they do not yet know based on recommendations of other agents. When facing the purchase of an item, agents query their neighbourhood for recommendations on the item to purchase. Neighbours in turn pass on a query to their neighbours in case that they cannot provide a reply themselves. In this way, the network replies to a query of an individual by offering a set of recommendations. One way to deal with these recommendations would be to choose the most frequently recommended item. However, because of the heterogeneity of preferences of agents, this may not be the most efficient strategy in terms of utility. Thus, we explore means to incorporate knowledge of trustworthiness of recommendations into the system. In the following, we investigate, by means of analytical calculations and computer simulations, under which conditions and to what extent the presence of a trust system enhances the performance of a recommendation system on a social network.

3.1 Agents, objects, and profiles

We consider a set S_A of N_A agents $a_1, a_2, a_3, \ldots, a_{N_A}$. The idea is that the agents are connected in a *social network* such as, for example, of people and their friends [3,28,31] that are recommending books to each other. Hence, each agent has a set of links to a number of other agents (which we call its neighbours). These links are not necessarily symmetric, i.e. the graph is directed. In reality, social networks between agents evolve over time; in other words, relationships form, sustain, and also break up. In this paper, we mainly focus on a static network while dynamic networks will be investigated more thoroughly in further work. At this stage, we assume the network to be described by a random graph [8,12]—the usual choice in absence of knowledge of the real structure of the modeled social network. We are aware that random graphs are not always a good approximation of real networks. Thus, for further analysis of the model, it will be appropriate to experiment with several different topologies as discussed in [2].

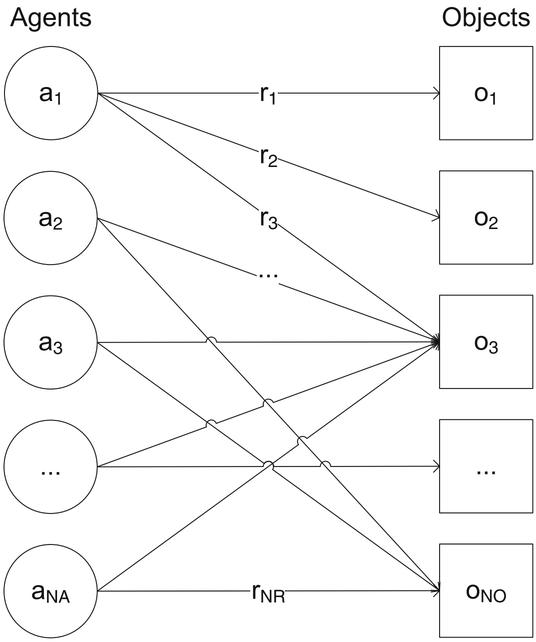
Furthermore, there exists a set S_O of N_O objects, denoted $o_1, o_2, o_3, \ldots, o_{N_O}$. These objects represent items, agents, products, buyers, sellers, etc.—anything that may be subject to the recommendations—i.e. books as in the running example. We further assume that objects are put into one or more of N_C categories from S_C, denoted $c_1, c_2, \ldots, c_{N_C}$, where these categories are defined by the system and cannot be modified (i.e. added, removed, or redefined) by the agents. In the scenario where the recommendation system is on books, categories could be ‘epicurean philosophy’, ‘Swiss folklore’, or ‘medieval archery’. We denote the fact that an object o_i is in category c_j by stating $o_i \in c_j$.

Each agent a_i is associated to one certain preference profile which is one of N_P preference profiles in the system, where $S_P = \{p_1, p_2, p_3, \ldots, p_{N_P}\}$. In the following, we will use the terms ‘preference profile’, ‘profile’, and ‘preferences’ interchangeably. Such a profile p_i is a mapping which associates to each object $o_j \in S_O$ a particular corresponding rating $r_j \in [-1, 1]$, $p_i : S_O \to [-1, 1]$. This is illustrated in Fig. 1. In the current version of the model, we only consider discrete ratings where -1 signifies an agents’ dislike of an object, 1 signifies an agents’ favour towards an object. In a future version of the model, this assumption can be relaxed; we chose to initially focus on a discrete rating scheme because most of the ones found on the Internet are of such type. We assume that agents only have knowledge in selected categories and, in particular, they do only know their own ratings on objects of other categories subsequent to having used these objects. Thus, each agent is and remains an expert only on a set of initially assigned selected categories.

3.2 Trust relationships

In this model, we also consider trust relationships between agents: each agent a_i keeps track of a trust value $T_{a_i,a_j} \in [0, 1]$ to each of its neighbour agents a_j. These values are initialised to $T_{a_i,a_j} = 0.5$. It is important to stress that trust relationships only exist between neighbours in the social network; if two agents are not directly connected, they also cannot possibly have a trust relationship with each other. However, two such agents may indirectly be connected to each other through a path in the network. For example, agent a_i could be connected to agent

Fig. 1 Agents rating objects: this is a bipartite graph with the agents on the left hand side and the objects on the right hand side, the ratings being the connections. The set of all possible ratings of an agent constitutes its respective profile



a_j through agents a_k and a_l, should a_k and a_l, a_i and a_k, as well as a_l and a_j be neighbours. We can then compute a trust value along the path path(a_i, a_j) from a_i to a_j—in the example, path$(a_i, a_j) = \{(a_i, a_k), (a_k, a_l), (a_l, a_j)\}$—as follows:

$$T_{a_i,\ldots,a_j} = \prod_{(a_k,a_l)\in \text{path}(a_i,a_j)} T_{a_k,a_l} \tag{1}$$

i.e. the trust value along a path is the product of the trust values of the links on that path. Of course, there may be more than one path between two agents; in such cases, each path has its own trust value. Figure 2 illustrates a part of such a social network of agents and a chain of trust relationships between two agents.

Note that this implies the assumption that trust is able to propagate through the network. In other words, we take the position that "if i trusts j, and j trusts k, then i should have a somewhat more positive view of k based on this knowledge" [19].

Trust transitivity is a condition for trust propagation and there have been fierce discussions in the literature whether or not trust is transitive. From the perspective of network security (where transitivity would, for example, imply accepting a key with no further verification based on trust) or formal logics (where transitivity would, for example, imply updating a belief store with incorrect, impossible, or inconsistent statements) it may make sense to assume that trust is not transitive [11,20,22]. Others attribute trust some degree of transitivity [18,19]. Furthermore, it has been shown empirically [14,15,19] that in scenarios similar to ours, it can be assumed that "trust may propagate (with appropriate discounting) through the relationship network" [19]. In our model, discounting takes place by multiplying trust values along paths.

It is important to remark that we do not allow recommendations by other agents to influence the preferences of an agent on items. Rather, recommendations are suggestions. An agent

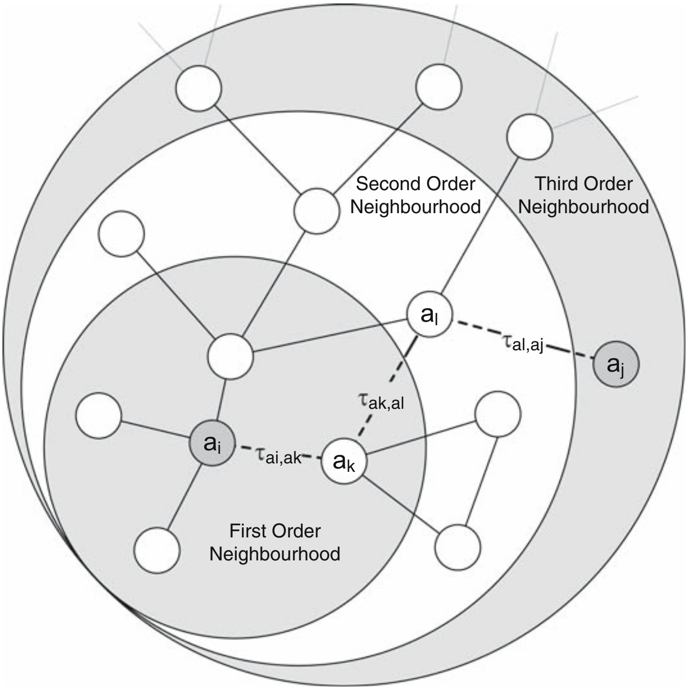


Fig. 2 Social network of agents and their trust relationships: a section of the social network around agent a_i, indicating a chain of trust relationships to agent a_j and ordering the neighbours according to their distance in hops ('orders of neighbourhood')

merely selects one and then, based on its experience, draws its own consequences, regardless of the recommendation.

3.3 Temporal structure, search for recommendations

The model assumes a *discrete linear bounded model of time*. In essence, there are two possible types of search for a recommendation:

1. *Ranking within a category* (*RWC*): agents query for a particular category and search recommendations for several objects in this category in order to decide for one of the recommended objects in the response from the network—typically the best one.
2. *Specific rating for an object* (*SRO*): agents query for a particular object and search recommendations on this very object in order to decide for or against using it, based on the response from the network.

Both variants are possible within the framework of our model: in fact, we use SRO to establish RWC.

At each time step t, each agent a_i (in random order) selects a category c_j (again, in random order, with the constraint that the agent is not an expert on the category) and searches for recommendations on the network. The protocol for agent's search proceeds as follows:

1. Agent a_i prepares a query(a_i, c_j) for category c_j and then transmits it to its neighbours.
2. Each neighbour a_k receives query(a_i, c_j) and either

(a) returns a response($a_k, a_i, (o_j, r_j), T_{a_i,\ldots,a_k}$), if it knows a rating r_j for a particular object o_j in c_j that it can recommend, i.e. if $p_k(o_j) = r_j > 0$ (only positive ratings are communicated, we do not consider negative ones at this stage for sake of simplicity in the decision making process)

(b) or, passes query(a_i, c_j) on to its own neighbours if it does not know a rating r_j for the particular category c_j.

Notice that an agent a_k knows the rating r_j for a particular object o_j only if it has either experienced it or if it is an expert on the category c_j of the object. Furthermore, each agent along the path computes only a part of the product $T_{a_i,\ldots,a_k}$—i.e. on the path path(a_1, a_2, a_3, a_4), agent a_3 would pass T_{a_3,a_4} to a_2 and then, a_2 would compute $T_{a_2,a_3,a_4} = T_{a_2,a_3} T_{a_3,a_4}$ and pass it to agent a_1 who then can compute $T_{a_1,a_2,a_3,a_4} = T_{a_1,a_2} T_{a_2,a_3,a_4}$.

It is assumed that agents keep track of the queries they have seen. Now there are two strategies to guarantee that the algorithm terminates: either,

- agents do not process queries that they have already seen again ("incomplete search", IS); or,
- agents pass on queries only once, but, if they have an appropriate recommendation, can return responses more than once ("complete search", CS).

In essence, both are a form of *breadth-first search* on the social network of agents, but with different properties: the former returns, for each possible recommendation, only one possible path in the network from the querying to the responding agent; the latter, however, returns, for each possible recommendation, each of the possible paths in the network from the querying to the responding agent.

As we will see later, this is a crucial difference for the decision-making of agents. For a given recommendation, there might be several paths between the querying and the responding agent. The IS returns a recommendation along one of these paths, while the CS returns a set of recommendations along all possible paths. Some paths between two agents have high trust, some have low trust. The IS may return a recommendation along a low-trust path even though there exists a high-trust path, thus providing an agent with insufficient information for proper decision-making. Of course, there is also a pitfall with the CS—it is computationally much more expensive. In the literature, this issue of potentially having multipe paths for a recommendation has been discussed [18], and we will come back to it when discussing the decision-making of the agents.

3.4 Decision-making

As a result of a query, each agent a_i possesses a set of responses from other agents a_k. It now faces the issue of making a decision for a particular object. The agent needs to decide, based on the recommendations in the response, what would be the appropriate choice of all the objects recommended. In the following, we denote query(a_i, o_j) $= Q$ and a response($a_k, a_i, (o_j, r_j), T_{a_i,\ldots,a_k}$) $\in R$ where R is the set of all responses. The values of trust along the path provide a ranking of the recommendations. There are many ways of choosing based on such rankings; we would like to introduce an exploratory behaviour of agents and an established way of doing so consists in choosing randomly among all recommendations with probabilities assigned by a logit function [32]. For this purpose, it is convenient to first map trust into an intermediate variable $\hat{T}$, ranging in $[-\infty, \infty]$:

$$\hat{T}_{a_i,\ldots,a_k} = \frac{1}{2} \ln \left(\frac{1 + 2(T_{a_i,\ldots,a_k} - 0.5)}{1 - 2(T_{a_i,\ldots,a_k} - 0.5)} \right) \in [-\infty, \infty] \tag{2}$$

i.e. $\hat{T}_{a_i,\ldots,a_k} = -\infty$ for $T_{a_i,\ldots,a_k} = 0$ and $\hat{T}_{a_i,\ldots,a_k} = \infty$ for $T_{a_i,\ldots,a_k} = 1$. Then,

$$P(\text{response}(a_k, a_i, (o_j, r_j), T_{a_i,\ldots,a_k})) = \frac{\exp(\beta \hat{T}_{a_i,\ldots,a_k})}{\sum_R \exp(\beta \hat{T}_{a_i,\ldots,a_l})} \in [0, 1] \tag{3}$$

where β is a parameter controlling the exploratory behaviour of agents (when $\hat{T}_{a_i,\ldots,a_k} = \pm\infty$, $P(\text{response}(a_k, a_i, (o_j, r_j), T_{a_i,\ldots,a_k}))$ is computed as a limit). With such transformations we achieve to have trust values $T_{a_i,\ldots,a_k}$ to lie in [0, 1] which is required in order to propagate them as well as negative values of $\hat{T}_{a_i,\ldots,a_k}$ when the trust towards an agent is very small—otherwise, agents would keep choosing recommendations even from untrustworthy agents with finite probability. For $\beta = 0$, the probability of choosing each response will be the same (i.e. this is equivalent to a random choice), but for $\beta > 0$, responses with higher associated values of $T_{a_i,\ldots,a_k}$ have higher probabilities. To decide for one of the objects, the agent chooses randomly between all recommendations according to these probabilities.

Now, suppose that an agent received a recommendation from another agent, but through many paths. For example, a_i may be linked to a_k through a_j, but also through a_l. Then, each of the two responses would be assigned a probability according to Eq. 3. Since recommendations coming along paths of high trust will have a higher probability of being chosen, this implies that recommendations coming along paths of low trust are still part of the decision-making process, but with much lower probability. This approach is similar to [18] (where only the highest path is considered, and all lower paths are discarded) and the issue has also been discussed in [22].

For benchmarking the trust-based approach of selecting recommendations, we consider an alternative decision-making strategy, namely a *frequency-based approach* without any trust relationships being considered at all. In this approach, an agent chooses randomly among each of the recommendations with equal probability for each of the recommendations.

3.5 Trust dynamics

In order to enable the agents to learn from their experience with other agents, it is necessary to feedback the experience of following a particular recommendation into the trust relationship. This is done as follows: subsequent to an interaction, agent a_i who has acted on a rating through its neighbour, agent a_j, updates the value of trust to this neighbour, based on the experience that he made. Let o_k be the chosen object. Then, assuming agent a_i having profile p_i, $p_i(o_k) = r_k$ is the experience that a_i has made by following the recommendation transmitted through a_j. It is convenient to define the update of $T(t+1)$ in terms of an intermediate variable $\tilde{T}(t+1)$:

$$\tilde{T}_{a_i,a_j}(t+1) = \begin{cases} \gamma \tilde{T}_{a_i,a_j}(t) + (1-\gamma) r_k & \text{for } r_k \geq 0 \\ (1-\gamma)\tilde{T}_{a_i,a_j}(t) + \gamma r_k & \text{for } r_k < 0 \end{cases} \tag{4}$$

where $\tilde{T}_{a_i,a_j}(0) = 0$ and $\gamma \in [0, 1]$. Because $\tilde{T}_{a_i,a_j} \in [-1, 1]$, we have to map it back to the interval [0, 1]:

$$T_{a_i,a_j}(t+1) = \frac{1 + \tilde{T}_{a_i,a_j}(t+1)}{2} \in [0, 1] \tag{5}$$

The distinction between $r_k \geq 0$ and $r_k < 0$ creates, for values of $\gamma > 0.5$, a slow-positive and a fast-negative effect which usually is a desired property for the dynamics of trust: trust is supposed to build up slowly, but to be torn down quickly. The trust update is only applied between neighbouring agents. Although the trust along pathways between two non-neighbour agents $T_{a_i,\ldots,a_j}$ is used for choosing a recommendation, this is not used to establish a value of trust towards non-neighbour agents. The trust along pathways between two non-neighbour agents $T_{a_i,\ldots,a_j}$ changes only as a result of changes on the links of the path, i.e. changes between neighbour agents.

Our intention is to keep the trust dynamics local, i.e. restrict it to neighbours. Any other approach would require agents to maintain global knowledge. The performance of the system results from the development of pathways of high trust and thus is an emergent property of local interactions between neighbouring agents.

It is important to note that—in the current version of the model— as a result of the trust dynamics, trust T_{a_i,a_j} evolves to a value which reflects the similarity of agents a_i and a_j. This is consistent with the observation in the literature that there is a correlation between trust and similarity [16] and that, in a recommendation system, "recommendations only make sense when obtained from like-minded people exhibiting similar taste" [34]. In fact, our mechanism could be seen as a possible explanation of this correlation. However, as stated, there are other interpretations of trust in different disciplines, in particular cognitive science, sociology, and psychology.

In further extensions of the model, trust T_{a_i,a_j} could include other notions such as "agent a_j cooperated with agent a_i", "agent a_j gave faithful information to agent a_i", or "agent a_j joined a coalition with agent a_i". In other words, T_{a_i,a_j} could be an aggregate of different dimensions of trust, possibly measuring the faithfulness, reliability, availability, and quality of advice from a particular agent.

3.6 Utility of agents, performance of the system

In order to quantitatively measure the difference of the trust-based approach of selecting recommendations as compared to the frequency-based approach, it is necessary to define measures for the utility of agents as well as for the performance of the system.

We define an instantaneous utility function for an agent a_i following a recommendation from agent a_j on object o_k at time t as follows:

$$u(a_i, t) = r_i \tag{6}$$

where agent a_i's profile determines $p_i(o_k) = r_i$. We consider the performance of the system to be the average of the utilities of the agents in the system:

$$\Phi(t) = \frac{1}{N_A} \sum_{a_i \in S_A} u(a_i, t) \tag{7}$$

This gives us a measure for quantitatively comparing the difference that the trust-based approach makes towards the frequency-based approach, both on the micro-level of an agent and the macro-level of the system.

4 Results

One of the most important results of the model is that the system self-organises in a state with performance near to the optimum. Despite the fact that agents only consider their own utility function and that they do not try to coordinate, long paths of high trust develop in the network, allowing agents to rely on recommendations from agents with similar preferences, even when these are far away in the network. Therefore, the good performance of the system is an emergent property, achieved without explicit coordination.

Three quantities are particularly important for the performance of the system: the network density, the preference heterogeneity among the agents, and the sparseness of knowledge. The core result is that recommendation systems in trust-based networks outperform frequency-based recommendation systems within a wide range of these three quantities:

- *Network density*: if the network is very sparse, agents receive useful recommendations on only a fraction of the items that they send queries about; the denser the network, the better the performance, but above a critical threshold for the density, the performance stabilises. The proximity of this value to the optimum depends on the other two quantities.
- *Preference heterogeneity*: if the preferences of agents are homogeneous, there is no advantage for filtering the recommendations; however, if the preferences of agents are all different, agents cannot find other agents to act as suitable filters for them. In between, when preferences are heterogeneous, but 'not too much', the system performance can be near to the optimum.
- *Knowledge sparseness*: when knowledge is dense (N_c and/or N_p small), it is easy for an agent to receive recommendations from agents with similar preferences. In the extreme situation in which, for each category there is only one expert with any given preference profile, agents can receive useful recommendations on all categories only if there exists a high-trust path connecting any two agents with the same profile. This is, of course, related to the density of links in the network.

The performance of the system thus depends, non-linearly, on a combination of these three key quantities. Under certain assumptions, the model can be investigated analytically and in a mean-field approach it is possible to make quantitative predictions on how these factors impact the performance. These results are presented in Sect. 4.1. In Sect. 4.2, we illustrate the properties of our recommendation system by describing the results of multi-agent simulations of the model. As a benchmark, we compare the trust-based recommendation system to a frequency-based recommendation system.

4.1 Analytical approximation

In the following, we derive an expression for the performance of the system as a function of the frequency and heterogeneity of profiles across agents. We proceed as follows. We first introduce the notion of similarity, ω of profiles. We then show, in the limit of a mean-field approximation, that the fix points of trust between two agents are a function of the similarity of their profiles. We then derive the value of the critical threshold for the network density above which a subset of agents with the same profile is expected to form a connected component. Above this threshold, agents with the same profile can receive recommendations on all categories covered by the expertise of such a subset of agents. Under this hypothesis, and in the stationary regime for the trust dynamics, the expected utility of an agent can easily be computed, again in a mean-field approximation, based on the decision-making dynamics used to choose among recommendations.

As common in the literature, the similarity between two profiles p_i, p_j is defined as

$$\omega_{i,j} = \frac{1}{N_O} \sum_{o_k \in S_O} 1 - \left| p_i(o_k) - p_j(o_k) \right| \tag{8}$$

The similarity of two agents is, for instance, 1 if their preferences over the products are identical, and -1 if their preferences over the products are always opposite, and 0 if half of their preferences are identical and half are opposite.

Suppose there are only two profiles p_1, p_2 in the population. If profiles are evenly distributed among agents ($n_1 = 1/2$), then the expected value of ω over a large set of pairs of profiles is $\langle\omega\rangle = 0$. If instead, agents have only one profile, $p_1 = 1$, then trivially $\langle\omega\rangle = 1$.

4.1.1 Trust dynamics

Consider the dynamics for the update of trust of agent a_i towards a neighbouring agent a_j (Eq. 4). Assume the two agents have profiles p_m and p_n, respectively (since the number of agents and of profiles are different, we don't use p_i and p_j for the profiles as this would suggest that there exactly as many profiles as agents). Let us then focus first only on recommendations coming directly from a_j. Equation 4 is a stochastic equation because recommendations are provided by a_j to a_i on objects of randomly chosen categories. In a mean-field approximation, we replace the stochastic term, r_k, with its average over time, which, by definition, is $\omega_{m,n}$. It is then straightforward to check that the fix point of both cases of Eq. 4 is $\omega_{m,n}$. By the time agent a_i has developed a value of trust towards a_j close to $\omega_{m,n}$, a_j has done the same with its own neighbours. In particular, if $\omega_{m,n}$ is close to 1, then only recommendations from neighbours a_w of a_j towards which a_j has developed high trust, are associated with high values of trust along the path a_w, a_j, a_i. The same holds, by induction, for longer paths. Therefore, we can extend the mean-field approximation also to the general case of recommendations received by a_i indirectly through a_j.

4.1.2 Random graph structure and critical density

It is known that in a random graph of N nodes and ℓ links, a *giant connected component* appears for values of $\ell > (N-1)/2$, meaning that the probability that the network is connected tends to 1 for large N (and correspondingly large ℓ) [8,12]. Equivalently, above this threshold, there is at least one path between any two randomly chosen nodes. In our model, agents are connected in a random graph and have different preference profiles, distributed randomly according to some frequency distribution. We can then ask what is the critical density of links (randomly drawn among agents of any profile) in the network such that there is (in the limit of many agents) at least one path between any two agents with the same profile. In this situation, a querying agent is able to receive recommendations from all other agents of the same profile along paths which involve only agents of the same profile. If n_i is the frequency of agents of profile p_i, we denote $\ell_{i,i} = \ell n_i^2$ to be the number of links among any two agents with same profile p_i. The condition for the existence of a giant component of agents with profile p_i is $\ell_{i,i} > ((N-1)/2)$, which implies $\ell > ((N-1)/(2n_i^2)$. For instance, for two profiles with frequency $n = 0.5$, this formula leads to $\ell = 2(N-1)$. In other words, the smaller the frequency of profile p_i, the higher the critical number of links ℓ above which agents with profile p_i become connected in a giant component.

4.1.3 Performance

As described in the decision-making process, at each time step, as a result of a query for a given category, an agent a_i receives a set of ratings associated with values of the trust along the paths from which the responses come from. Each rating is selected with a probability given by Eq. 3. Over time, the agent sends many queries. Let R be the set of all responses k it receives over time. The expected value of the rating r, hence of the utility u of the agent, is then:

$$E(u) = E(r) = \sum_{k \in R} r_k P_k = \frac{\sum_{k \in R} r_k \exp(\beta \hat{T}_k)}{\sum_{k \in R} \exp(\beta \hat{T}_k)} \tag{9}$$

The set R contains responses sent by many agents with different profiles. We can group the set R by the set S_p of profiles of such agents. In a mean-field approach we can then replace the ratings experienced by the agent a_i, with profile p_q, through following recommendations of other agents with profile p_s, with its average value $\omega_{q,s}$. In the same spirit, we can also replace the value of trust towards all agents with profile p_s by $\omega_{q,s}$. Then, Eq. 2 implies that $\exp(\beta \hat{T})$ is approximated with the value $(1+\omega)/(1-\omega)^{\frac{\beta}{2}}$. This approximation is well-justified for first neighbours. For the other agents, it is less accurate, but it may be expected to hold if the network is well above the density threshold and in the stationary regime in which trust paths have already developed. The expected utility of an agent, which, in the long run, coincides with the expected value of the performance of the system, is then:

$$E(\Phi) = \frac{\sum_{s \in S_p} \omega_{q,s} (\frac{1+\omega_{q,s}}{1-\omega_{q,s}})^{\frac{\beta}{2}}}{\sum_{s \in S_p} (\frac{1+\omega_{q,s}}{1-\omega_{q,s}})^{\frac{\beta}{2}}} = \frac{\sum_{\omega} \omega (\frac{1+\omega}{1-\omega})^{\frac{\beta}{2}} \nu(\omega)}{\sum_{\omega} (\frac{1+\omega}{1-\omega})^{\frac{\beta}{2}} \nu(\omega)} \tag{10}$$

where the second expression is obtained as follows: we group the set S_p by the values of similarity between the profile of the querying agent and the profiles of the recommending agents. Since, in a pair of querying-responding agents there is a finite number of combinations of profiles, and their probability of occurrence depends the relative frequency of each profile in the population (profiles are assigned randomly to the agents). Therefore, the probability of occurrence of each value of similarity ω, $\nu(\omega)$ is known by construction. Each term $(1+\omega)/(1-\omega)^{\frac{\beta}{2}} \nu(\omega)$ represents the probability of an agent choosing the recommendation from an other agent with a given similarity value ω, multiplied by the probability that such a similarity value occurs among the querying agent and the recommenders. This formula allows to predict the expected utility of the system as a function of the distribution of the profiles of preferences among the agents. The formula holds in the regime in which each subset of agents of the same profile form a connected component and their joint expertise covers all the categories. For instance, if we consider two profiles in the system p_1 and p_2, with frequency n_1 and $1-n_1$, the probability that a pair of agents consists of both p_1, or both p_2, or mixed is, respectively: $(n_1)^2$, $(n_2)^2 = (1-n_1)^2$ and $2(n_1)(1-n_1)$. The corresponding values of ω are $1, 1, -1$.

In the absence of trust (i.e., $\beta = 0$), Eq. 10 reduces to the expression of the expected value of ω, yielding $\Phi = 4n_1^2 - 4n_1 + 1$. In presence of trust (i.e., $\beta > 0$), the term with ω close to 1 dominates, thus yielding $\Phi \approx 1$. These results will be confirmed empirically in the next section.

4.2 Computer simulations

For the simulations we have used the following parameters to the model: we consider $N_a = 100$ agents, and the simulations are averaged over $N_r = 100$ runs. The size of each category is the same and we vary $N_c \in \{10, \ldots, 50\}$ and $N_p \in \{2, 4, 6\}$; N_o is usually adjusted such that there are at least two objects in each category. Profiles are distributed such that the sum over a profile is 0 on average—across the profile, categories, and agents. Each agent is an expert on 1 category. Further, for the social network we assume a random directed graph with a given number of agents, N_a, and a given total number of links, ℓ. The *network density*

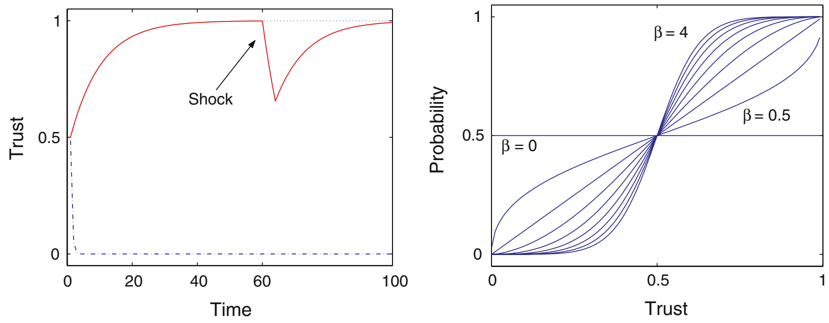


Fig. 3 Dynamics of trust and logit function. Left: slow-positive fast-negative dynamics of trust. Trust between two agents of the same profile (black dotted line), between two agents of opposite profiles (blue dashed line). In case that an agent recommends an object that is rated negatively, trust drops quickly and recovers slowly (red solid line). Right: impact of the choice of the exploration parameter β on the decision-making. The slope of the sigmoid-like function increases for increasing values of β

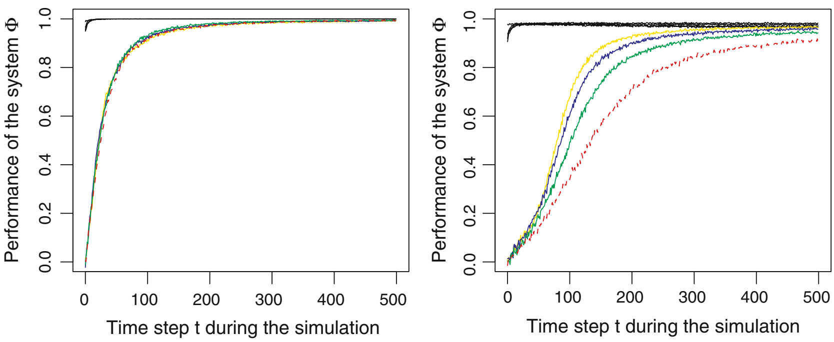


Fig. 4 Performance versus Time for $N_c = 10$ (left) and $N_c = 50$ (right). Over time, performance approaches the optimum—with learning (top black line), this process is accelerated. Different curves correspond to different values of $\gamma \in \{0.2, 0.4, 0.6, 0.8\}$. Increasing values of γ lead to curves approaching the optimum faster (corresponding colours: red, green, blue, yellow)

is then defined as $d = \ell/N_a(N_a - 1)$. Agents are connected randomly with respect to their profile.

Figure 3 (left) shows that the update rule of trust as described by Eqs. 4 and 5 produces the desired slow-positive fast-negative dynamics. Trust between two agents of the same profile evolves to 1 (black dotted line, partially covered by the red solid line). Trust between two agents of opposite profiles evolves to 0 (blue dashed line). In case that an agent recommends an object that is rated negatively, trust drops quickly and recovers slowly (red solid line). The probability of choosing a recommendation depends critically on the parameter β, which controls the exploratory behaviour of agents, as shown in Fig. 3 (right).

Over time, each agent develops a value of trust towards its neighbours which reflects the similarity of their respective profiles. After some time, paths of high trust develop, connecting agents with similar profiles. As a result, the performance of the system, as defined in Eq. 7 increases over time and reaches a stationary value which can approach the optimum, as shown in Fig. 4, where the curves correspond to different values of γ. Increasing values of γ lead to curves approaching the optimum faster.

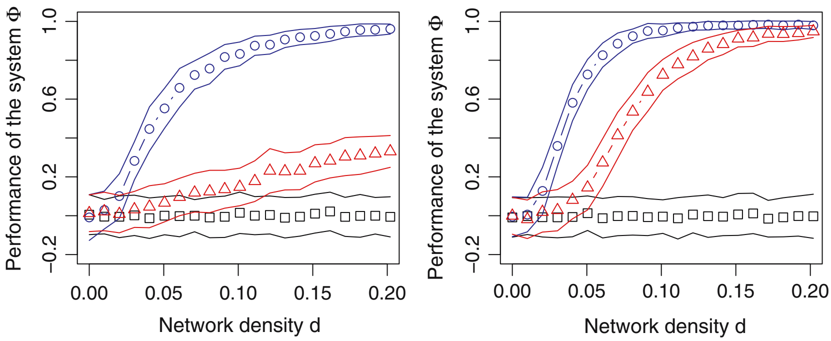


Fig. 5 Performance versus density for different N_c (in blue circles and red triangles, 10 and 50 categories, respectively). Incomplete search (left) and complete search (right). For sparse knowledge, the complete search performs much better than the incomplete search. The benchmark of a frequency-based system is represented by black squares

We have also simulated a situation in which, prior to the start of the dynamics, there is a learning phase in which the agents explore only the recommendations of their direct neighbours on the categories that these claim to be expert on. This way, the trust dynamics already start from a value deviating from the neutral point of 0.5 and closer to one of the fix points (see Eq. 4). In this case, the performance is optimal from the beginning on (top black curve). Interestingly, the system evolves, even in the normal dynamics, to the same value that is reached with the learning phase, supporting the idea that the optimal performance is an emergent behaviour of the system.

In the model description, we have described two types of search. Fig. 5—the performance Φ of the system plotted against increasing values of density d in the network—shows that the search type becomes important when the knowledge is sparse. We notice a sigmoid shape which would become steeper for systems with larger numbers of agents. We consider different N_c, corresponding to levels of sparseness of knowledge (in blue circles and red triangles, 10 and 50 categories, respectively, $N_p = 2$). With the incomplete search algorithm, the performance deteriorates. With the complete search algorithm, the system reaches the optimal performance even in the case of maximally sparse knowledge (50 categories means that there is only 1 expert from each profile in each category). In both plots of Fig. 5, the black squares correspond to the frequency-based recommendation system used as benchmark. In fact, without trust, the performance is 0 on average, because random choices lead to an equal distribution of “good” and “bad” objects (with respect to profiles).

We now illustrate the role of preference heterogeneity. We consider first the case in which there are two possible, opposite, profiles in the population, say p_1 and p_2. We define the fraction of agents characterised by the first profile as n_1. In Fig. 6 (left), we plot the performance of the system with and without trust (red triangles and blue squares, respectively) against increasing values of n_1. When $n_1 = 0.5$ there is an equal frequency of both profiles, while when $n_1 = 1$ all agents have the first profile. For the system without trust, the performance increases for increasing n_1. In fact, despite that choices are random, agents receive recommendations which are more and more likely to match the preferences of the majority. On the other hand, the minority of agents with the profile p_2 are more and more likely to choose wrong recommendations, but their contribution to the performance of the system decreases. The simulation results are in good agreement with the predictions obtained in the analytical approximation (black dotted lines), Eq. 10. For the system with trust the

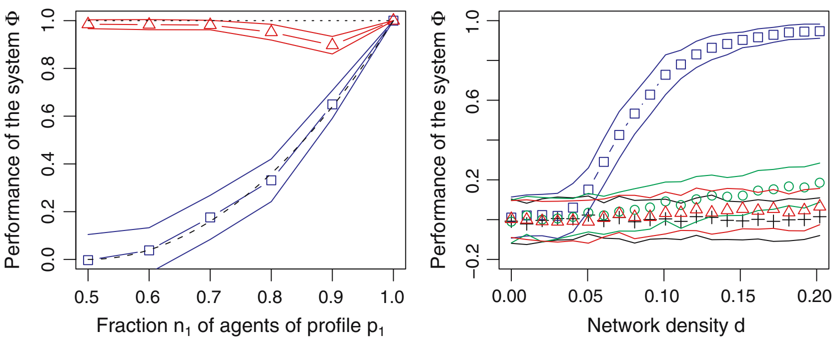


Fig. 6 Effect of heterogeneity on performance. The trust-based approach performs well also in very homogeneous systems; in the extreme case of very heterogeneous systems, performance drops. Left: performance as a function of the fraction n_1 of agents with profile p_1, with trust (red triangles) and without trust (blue squares). Right: performance as a function of network density d with different numbers of profiles N_p (blue squares: $N_p = 2$, green circles: $N_p = 4$, red triangles: $N_p = 6$). The benchmark of a frequency-based system is represented by black crosses

performance is almost unchanged by the frequency. This very strong result has the following explanation: the social network is a random graph in which agents have randomly assigned profiles. Agents assigned to p_2 decrease in number, but, as long as the minority, as a whole, remains connected (there is a path connecting any two such agents) they are able to filter the correct recommendations. At some point the further assignment of an agent to p_1 causes the minority to become disconnected and to make worse choices. In the simulations, this happens when $n_1 = 0.9$ and $n_2 = 0.1$. Another way of investigating the role of heterogeneity of preferences is to consider an increasing number of profiles in the population, each with the same frequency. In the extreme case in which, for each category there is only one expert with any given preference profile, the performance, at constant values of network density d, drops dramatically, as shown in Fig. 6 (right).

5 Extensions

So far, we have made the assumptions that (1) agents are self-interested in the sense of bounded rationality, but do not act randomly, selfishly, or maliciously and that (2) the social network of agents is fixed and does not change over time—no agents join or leave the networks and no links are rewired, added, or dropped. In reality, both of these assumptions need to be relaxed, so in further work, we plan to investigate the behaviour of the system with respect to these issues.

5.1 Evolving social network

Considering a fixed network between agents does not appropriately depict reality; usually, *social networks evolve over time* with links being created and deleted at each time step. For example, the network could evolve in the following manner: at certain intervals over time, each agent a_i randomly picks one of its links—e.g., to agent a_j—and rewires it to a random other agent in the network or keeps it, both with a certain probability. Of course, it would make sense to tie this probability to the level of trust the agent has on the particular link

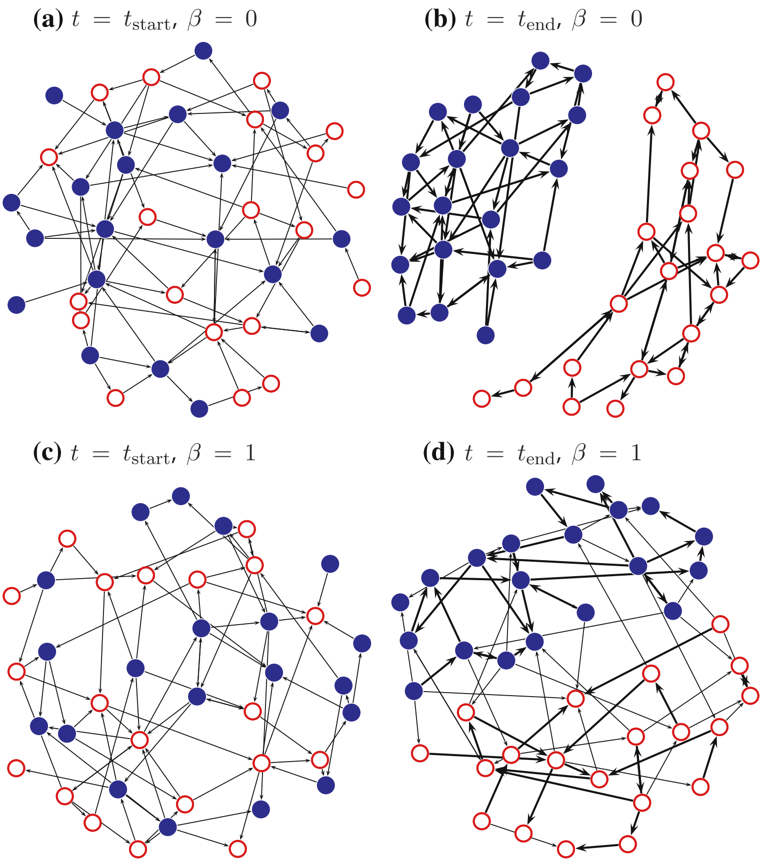


Fig. 7 Snapshots of the evolution of a network of 40 agents in two profiles and 80 links at time $t = t_{\text{start}}$ and $t = t_{\text{end}}$ for $\beta = 0$ and $\beta = 1$, respectively. When $\beta = 0$, disconnected clusters of agents with the same profile form, when $\beta = 1$, interconnected clusters of agents with the same profiles form. For $\beta > 0$, agents develop stronger ties to agents of the same profile than to agents of different profiles

considered for rewiring:

$$P(\text{rewire}) = 1 - T_{a_i,a_j} \tag{11}$$

$$P(\text{keep}) = T_{a_i,a_j} \tag{12}$$

i.e. $P(\text{rewire}) + P(\text{keep}) = 1$.

Figure 7 shows how snapshots of the evolution of a sample network of agents at different stages for different values of β look when applying this mechanism. Note the random graph structure at $t = t_{\text{start}}$ and the community fragmentation at $t = t_{\text{end}}$. This illustrates the dilemma between exploration and exploitation faced by the agents. For $\beta = 0$, agents choose randomly, thus performing worse, but they explore many of the other agents repetitively and their trust relationships converge to the steady state of the trust dynamics of Eqs. 4 and 5. Then, over time, links with low trust are rewired and links with high trust are kept. This leads to the emergence of two disconnected clusters. Eventually, subsequent to the formation of clusters, such agents will perform well, as any recommendation will come from an agent of the same profile. For $\beta = 1$, agents choose according to the strength of trust relationships, thus performing better, and they are able to exploit their knowledge. However, they exploit stronger links while not even exploring weaker ones. This results in clustering, but with interconnections between clusters. As networks in reality are evolving, it is important to study the impact of such behaviour on the system in more detail.

5.2 Robustness against Random, Selfish, and Malicious Agents

Another extension of the model focuses on the robustness of the recommendation system against attacks. For this purpose, three different additional types of agents can be considered: (1) *Random agents* are agents that, instead of giving correct recommendations, give a random recommendation. Having such agents in the system mimics the effect of noise on communication channels. (2) *Selfish agents* are agents that do not return recommendations except in the case that they have already received responses through the agent that initiated the query. (3) *Malicious agents* are agents that intentionally give recommendations that do not correspond to their own beliefs—i.e., they recommend what they would not use themselves, and vice versa. We are interested in the performance of the recommendation system with respect to differing fractions of such agents in the system: To what extent is the performance affected? Is there a critical value of the fraction of such agents for which the recommendation system becomes unusable? For applications in reality, an analysis of these topics is crucial.

6 Summary and conclusions

We have outlined a model for a trust-based recommendation system that combines the concepts of social networking and trust relationships: agents use their trust relationships to filter the information that they have to process and their social network to reach knowledge that is located far from them. Probably the most striking result of this work is that the recommendation system self-organises in a state with performance near to the optimum; the performance on the global level is an emergent property of the system, achieved without explicit coordination from the local interactions of agents. With this model, we strive towards building an archetypal model for recommendation systems by combining the concepts of social networking and trust relationships.

References

1. Abdul-Rahman, A., & Hailes, S. (2000). Supporting trust in virtual communities. In *Proceedings of the 33th Annual Hawaii International Conference on System Sciences*, IEEE Press.
2. Amaral, L. A. N., Scala, A., Barthélémy, M., & Stanley, H. E. (2001). Classes of small-world networks. *Proceedings of the National Academy of Sciences 97*, 21, 11149–11152.
3. Barabasi, A.-L., & Albert, R. (1999). Emergence of scaling in random networks. *Science, 286*, 509–512.
4. Battiston, S., Bonabeau, E., & Weisbuch, G. (2003). Decision making dynamics in corporate boards. *Physica A, 322*, 567.
5. Battiston, S., & Catanzaro, M. (2004). Statistical properties of board and director networks. *European Journal of Physics B, 38*, 345–352
6. Battiston, S., Walter, F. E., & Schweitzer, F. (2006). Impact of trust on the performance of a recommendation system in a social network. In *Proceedings of the Workshop on Trust in Agent Societies at the Fifth International Joint Conference on Autonomous Agents and Multi-Agent Systems (AAMAS'06)*.
7. Battiston, S., Weisbuch, G., & Bonabeau, E. (2003). Spread of decisions in the corporate board network. *Advances in Complex Systems 6*(4), 631–644.
8. Bollobás, B. (1985). *Random graphs*. Academic Press.

9. Brin, S., & Page, L. (1998). The anatomy of a large-scale hypertextual Web search engine. *Computer Networks and ISDN Systems, 30*(1–7), 107–117.
10. Castelfranchi, C., & Falcone, R. (2001). Social trust: a cognitive approach. *Trust and deception in virtual societies*. Kluwer Academic Publishers, pp. 55–90.
11. Christianson, B., & Harbison, W. S. (1997). Why isn't trust transitive? In *Proceedings of the International Workshop on Security Protocols*, pp. 171–176, Springer Verlag.
12. Erdős, P., & Rényi, A. (1959). On random graphs. *Publicationes Mathematicae Debrecen, 6*, 290–291.
13. Garlaschelli, D., Battiston, S., Castri, M., Servedio, V. D. P., & Caldarelli, G. (2003). The scale-free topology of market investments. *Physica A, 350*(2-4), 491–499.
14. Golbeck, J. (2006). Combining provenance with trust in social networks for semantic web content filtering. In *Revised Selected Papers of the International Provenance and Annotation Workshop (IPAW 2006)*, Moreau, L., & Foster, I. T. (Eds.), *Lecture notes in computer science*. (Vol. 4145, pp. 101–108). Springer.
15. Golbeck, J. (2006). Generating predictive movie recommendations from trust in social networks. In *Proceedings of the 4th International Conference on Trust Management (iTrust 2006)*, Stølen, K., Winsborough,s W. H., Martinelli, F., & Massacci, F. (Eds.), *Lecture notes in computer science*, (Vol. 3986, pp. 93–104). Springer.
16. Golbeck, J. (2006). Trust and nuanced profile similarity in online social networks. forthcoming.
17. Grandison, T., & Sloman, M. (2000). A survey of trust in internet applications. *IEEE Communications Surveys and Tutorials, 3*(4), 2–16.
18. Gray, E., Seigneur, J.-M., Chen, Y., & Jensen, C. D. (2003). Trust propagation in small worlds. In *Proceedings of the 1st International Conference on Trust Management*, Nixon, P. & Terzis, S. (Eds.), *Lecture Notes in computer science*, (Vol. 2692, pp. 239–254). Springer.
19. Guha, R., Kumar, R., Raghavan, P., & Tomkins, A. (2004). Propagation of trust and distrust. In *WWW '04: Proceedings of the 13th International Conference on the World Wide Web*, pp. 403–412. ACM Press,
20. Huang, J., & Fox, M. S. (2006). An ontology of trust: formal semantics and transitivity. In *Proceedings of the 8th International Conference on Electronic Commerce (ICEC '06)*, pp. 259–270. ACM Press.
21. Huberman, B. A., & Adamic, L. A. (1999). Growth dynamics of the World-Wide Web. *Nature, 401*, 131.
22. Jøsang, A., & Pope, S. (2005). Semantic constraints for trust transitivity. In *Proceedings of the 2nd Asia-Pacific Conference on Conceptual Modelling (APCCM2005)*, Hartmann, S., & Stumptner, M. (Eds.), APCCM (Vol. 43, pp. 59–68). Australian Computer Society.
23. Marsh, S. (1994). *Formalising trust as a computational concept*. PhD thesis, University of Stirling.
24. Massa, P., & Bhattacharjee, B. (2004). Using trust in recommender systems: An experimental analysis. In *Proceedings of the 2nd International Conference on Trust Management (iTrust 2004)*, Jensen, C. D., Poslad, S., & Dimitrakos, T. (Eds.), *Lecture notes in computer science*. (Vol. 2995, pp. 221–235). Springer,
25. Montaner, M., López, B., & de la Rosa, J. L. (2002). Opinion-based filtering through trust. In *Proceedings of the 6th International Workshop on Cooperative Information Agents (CIA 2002)*, pp. 164–178 Springer.
26. Montaner, M., López, B., & de la Rosa, J. L. (2003). A taxonomy of recommender agents on the internet. *Artificial Intelligence Review, 19*(4), 285–330.
27. Mui, L., Mohtashemi, M., & Halberstadt, A. (2002). A computational model of trust and reputation for E-businesses. In *Proceedings of the 35th Annual Hawaii International Conference on System Sciences*, IEEE Press.
28. Newman, M. E. J. (2003). The structure and function of complex networks. *SIAM Review, 45*(2), 167–256.
29. Sabater, J., & Sierra, C. (2005). Review on computational trust and reputation models. *Artificial Intelligence Review 24*(1), 33–60.
30. Sarwar, B., Karypis, G., Konstan, J., & Riedl, J. (2000). Analysis of recommendation algorithms for E-commerce. In *Proceedings of the 2nd ACM Conference on Electronic Commerce (EC-2000)*, pp. 158–167. ACM Press.
31. Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *Nature, 393*, 440–442.
32. Weisbuch, G., Kirman, A., & Herreiner, D. (2000). Market organisation and trading relationships. *The Economic Journal, 110*, 411–436.
33. Wellman, B. (2001). Computer networks as social networks. *Science, 293*, 2031–2034.
34. Ziegler, C.-N., & Golbeck, J. (2006). Investigating correlations of trust and interest similarity. *Decision Support Systems 43*(2), (forthcoming).